# A Multiple-Sweep-Frequencies Scheme Based on Eigen-decomposition to Compensate Ionospheric Phase Contamination

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*Abstract*—**This paper presents an improved scheme to compensate nonlinear phase path contamination when the backscattered signal propagates through the ionosphere in high-frequency skywave radar systems. The ionospheric variation often causes the spread of the ocean clutter spectrum in the frequency domain. The energy of the first-order component of ocean clutter dominates in Doppler spectrum, and thus its spreading may submerge the neighboring low-velocity target easily. The instantaneous frequency (IF) estimating algorithm based-on eigen-decomposition has been introduced to estimate the frequency fluctuation due to ionospheric phase path variation and the compensation is carried out before the coherent integration. In the proposed multiple-sweep-frequencies scheme we construct a new "sweep-frequency" dimension by using different transmitting frequencies, and because of the different variation of Doppler frequencies for the target echo and ocean clutter first-order Bragg lines with the varying transmitting frequencies, that can be considered as different "sweep-frequency angles", the full-rank autocorrelation matrix used in eigen-decomposition can be formed. Better estimation accuracy is achieved and significant spectral sharpening can be observed in the resultant spectrum. To avoid the additional systemic complexity due to the multiple frequencies sweep operation, a segmenting range transform in an assistant channel is proposed to obtain the 'sweep-frequencies' dimension data and estimate the ionospheric contamination. Experiments show that the proposed scheme is effective and its performance is discussed.**

*IndexTerms*—**High-Frequency Skywave Over-the-horizon radar, multiple-sweep-frequencies, eigen-decomposition, phase path variation**

## I. INTRODUCTION

High-frequency skywave over-the-horizon radar (OTHR) can provide a range-coverage of up to 4,000km by means of the refraction within the ionosphere. But the signal contamination suffered in double ionospheric transit. Especially, the Doppler spread mechanism renders the ocean clutter spectrum distorted and the resolution of coherent integration technique is degraded extremely [1]-[3].

The echo signal reflected by the sea surface has a pair of peaks in the Doppler domain, which is known as the Bragg lines. In some applications of HF skywave radar such as remote sensing and sea surface surveillance, temporal nonlinear phase path variation often produces significant spectral spread in the Doppler frequency domain so that the Bragg lines and target echoes often smear cross. Since the energy of the Bragg lines is much stronger that slightly spreading first-order ocean clutter spectrum can obscure the neighboring echo scattered by a slow moving surface vessels. The phase contamination may be attributed to several complex geophysical mechanisms. Effective frequency management system can release this problem to some extent, but it is not always feasible. A short dwell time can also be used to weaken the effect of the ionosphere at the price of lower frequency resolution [4].

To limit the Doppler spread effect and allow extended coherent integration time for good frequency resolution, it is necessary to estimate and compensate the raw radar signal by signal processing techniques before coherent integration. By virtue of the non-stationarity of the phase perturbation, some instantaneous frequency estimation methods are introduced to solve this problem. Bourdillon *etc.* have suggested correcting the phase of the signal from a perturbation estimated using maximum entropy spectrum analysis (MESA) as an estimator for the quasi-instantaneous frequency of the Bragg lines with time [5]. However, it is likely to fail for the contamination with periods shorter than a few tens of second integration time used in the autoregressive spectral estimation process. Parent and Bourdillon proposed a simpler technique using time derivative of the signal phase as the estimation of the instantaneous frequency of the contamination with short periods [6,7]. This method has introduced first the idea of multiple operating frequencies but is not suitable for regular FMCW radar because of its complexity.

Abramovich, Anderson, and Soloman addressed another method based on eigenvalue decomposition technique [8]. It is an advanced version of noise subspace technique applied to the instantaneous frequency estimation. However, since the correlation between the adjacent range, azimuth, and frequency bins of the echo signal is unknown and varying, the autocorrelation matrix estimated from the available clutter data set may be biased or rank-deficient, that means the eigen-decomposition technique under these conditions is defective or even ineffective. In this paper, a new "multiple-sweep-frequencies" scheme derived from the above eigen-decomposition method is proposed to solve this problem.

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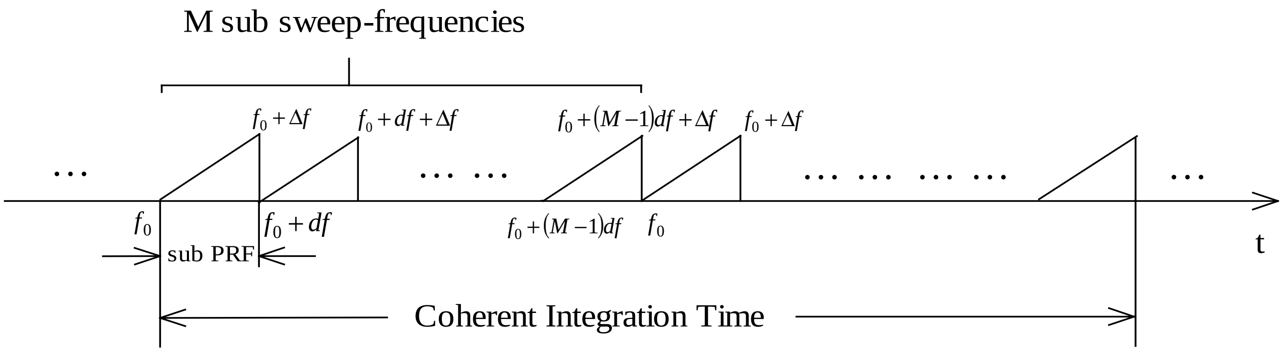



Fig.1. Transmitting signal form of the multiple-sweep-frequencies scheme

## II. MULTIPLE-SWEEP-FREQUENCIES SCHEME

As one of primary spectrum analysis methods, subspace technique can provide considerable spectrum resolution and hence is widely applied to various fields. A number of algorithms based on subspace decomposition have been developed [9]. In this section, we apply a multiple-sweep-frequencies scheme to achieve good estimation of the autocorrelation matrix that is used in the eigen-decomposition algorithm.

The transmitting signal form is designed as Fig.1. We divide the whole sweep frequency bandwidth into M sub-intervals, and a new dimension named as "sweep-frequency" dimension consists of the sub-intervals. Each sweep frequency sub-interval starts at slightly different operating frequency. The starting frequency spacing between two neighboring sub-intervals in a whole sweep frequency interval is $df$ , where $df \ll f_0$ . The sub-sweep-frequency bandwidth is $\Delta f$ , and in all sub-intervals the sweep frequency rates are identical.

Usually the received signal of the sensors can be expressed by

$$Y = D\left(\vec{S}\vec{A}' + W\right) \tag{1}$$

where $\vec{S}$ is N-element observed signal vector, $\vec{A}$ is M-element "sweep-frequency angle" vector, $W$ is $N \times M$ noise matrix, M is the sub-sweep-frequency number, and N is the number of sweep frequency period in a coherent integration period. It should be noted that only single mode propagation is discussed in this paper. The phase contamination modulation matrix $D$ is defined by

$$D = \begin{bmatrix} \ell^{j\phi(1)} & 0 & \cdots & 0 \\ 0 & \ell^{j\phi(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \ell^{j\phi(N)} \end{bmatrix} \tag{2}$$

where $\phi(n)$ is the phase contamination sequence and $1 \le n \le N$ .

In the classical ocean clutter theorem by Barrick [10], the first-order Bragg lines are located in the Doppler frequency domain

$$f_B = \pm\sqrt{\frac{gf_0}{\pi c}} \tag{3}$$

where $g$ is the acceleration due to gravity, $c$ is the speed of light and $f_0$ is the radar operating frequency in Hz. In the multiple-sweep-frequency scheme, the neighboring sub-sweep-frequency periods would have different operating frequencies, in other word, have different ocean clutter Bragg Doppler frequencies in echo spectrum. The starting sub-frequency of the k-th sub-sweep-frequency period $f_{0,k}$ is given by

$$f_{0,k} = f_0 + (k-1)df \tag{4}$$

where $1 \le k \le M$ . From (3) and (4), the Bragg Doppler frequency of the k-th sub-sweep-frequency period $f_{B,k}$ is given by

$$\begin{aligned} f_{B,k} &= \pm\sqrt{\frac{gf_{0,k}}{\pi c}} = \pm\sqrt{\frac{g\left(f_0 + (k-1)df\right)}{\pi c}} \\ &= \pm\sqrt{\frac{gf_0}{\pi c}} \cdot \sqrt{1 + (k-1)\frac{df}{f_0}} \end{aligned} \tag{5}$$

Since $df / f_0 \ll 1$, an approximation can be achieved using the Taylor expansion expression

$$\sqrt{1 + (k-1)\frac{df}{f_0}} \approx 1 + \frac{1}{2}(k-1)\frac{df}{f_0} \tag{6}$$

Hence the "sweep-frequency angle" vectors of the Bragg lines can be derived

$$\vec{A}_B = \left[1, \ell^{j2\pi\left(\frac{\Delta}{2}\right)t}, \ell^{j2\pi\Delta t}, \ldots\ldots, \ell^{j2\pi\left(\frac{(M-1)\Delta}{2}\right)t}\right] \tag{7}$$

where $\Delta = df / f_0$ .

Similarly, the "sweep-frequency angle" vector of the target echo with the sub-sweep-frequency $f_{0,k}$ is given by

$$\vec{A}_T = \left[1, \ell^{j2\pi\Delta t}, \ell^{j2\pi(2\Delta)t}, \ldots\ldots\ldots, \ell^{j2\pi(M-1)\Delta t}\right] \tag{8}$$

Thus substituting $\vec{A}$ in (1) with $\vec{A}_B$ and $\vec{A}_T$ , we can estimate the autocorrelation matrix directly by

$$\hat{R} = \frac{1}{M} Y^H Y = \frac{1}{M}\left(D\left(\vec{S}_B \vec{A}_B + \vec{S}_T \vec{A}_T\right) + W\right)^H \cdot \left(D\left(\vec{S}_B \vec{A}_B + \vec{S}_T \vec{A}_T\right) + W\right) \quad (9)$$

where $\vec{S}_B$ is N-element ocean clutter signal vector and $\vec{S}_T$ is N-element target echo vector. Since the "sweep-frequency angles" of Bragg lines of ocean clutter and ship target echo are different, i.e. there are two independent single tone signals in "sweep-frequency" dimension, the unbiased estimate of autocorrelation matrix can be achieved by the covariance method and its rank is full.

To avoid this additional systemic complexity and make this multiple frequencies sweep scheme feasible, we use an assistant channel to estimate the ionospheric contamination and compensate it in main channel. Here a regular FMCW waveform is used in both channels.

In the assistant channel, a segmenting range transform is applied to replace the conventional range transform for constructing the 'sweep-frequency' dimension. The data before range processing can be divided into $M$ subsets, where $M$ is the size of 'sweep-frequency' dimension. For each subset, the classical range transform is performed by a FFT operation. Picking up the $p$ -th sample of each subset after range transform, where $p$ is the range bin of interest, we can achieve a $N \times M$ data matrix. This data matrix has similar properties of the multiple transmitting frequencies signal waveform mentioned above. Therefore, the multiple-sweep-frequencies scheme can be realized without any modification in transmitting and receiving subsystems of the existing FMCW radar.

Although the range resolution resulted from the above processing stage is lower, if the assumption that the ionospheric variation is strongly dependent among the adjacent range bins holds, the contamination information can be contained completely in the dataset processed with the above method.

## III. Estimation and compensation of the phase contamination

In this section, a process to estimate and remove the ionospheric nonlinear phase path contamination is addressed. Under the assumption that the phase contamination function in a few repetition periods is constant, we consider that any two sub-intervals in the time domain, consisting of these few repetition periods, experience the frequency shift modulated by the phase perturbation. So the frequency shift, also regarded as the instantaneous frequency of the perturbation, can be estimated by eigen-decomposition technique.

The first step of the estimation process is to determine if the echo signal was phase-modulated (or contaminated) by the ionosphere. In Reference [8], a method using the time-reversibility of the autocorrelation matrix was proposed. If the signal that is not distorted in the ionospheric propagation channel, its autocorrelation matrix retains the time-reversibility; otherwise, it loses. An alternative is to determine if the Bragg lines are at the known fixed frequency, known as the Bragg frequencies. If the Bragg lines are not at the expected frequency then the ionosphere must have changed and caused a phase contamination, and the estimation and correction process should be applied.

Once the fact that the received signal is contaminated is determined, a subspace dimension criterion to determine the rank of the autocorrelation matrix should be applied. As one of the best popular criteria, Akaike Information Criterion (AIC) [9] is used in this experiment.

The received echo of the k-th sub-sweep-frequency period, containing the ocean clutter and the target, is divided to Q sub-intervals, which has $N/Q$ samples. From the equation (9), the autocorrelation matrix of the i-th sub-interval can be estimated by

$$R_i = \frac{1}{M} Y_i^H Y_i, \qquad i = 1,2,......Q \qquad (10)$$

where $R_i$ is a $N/Q \times N/Q$ matrix. Then we can carry out eigen-decomposition of the autocorrelation matrix $R_i$ and define the signal and noise subspace $\ddot{S}_i$ , $\ddot{G}_i$ . A frequency difference finding matrix is defined as

$$D(\omega) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ell^{j\omega T_R} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \ell^{j\omega (N/Q-1) T_R} \end{bmatrix} \qquad (11)$$

where $T_R$ is the interval between two neighboring sub-intervals. Then we construct an orthogonal error sum function

$$\mu_i(\omega) = \sum_{n=1}^{N/Q-r} \sum_{m=1}^{r} \vec{S}_{m,i+1}^H D(\omega) \vec{G}_{n,i} \vec{G}_{n,i}^H D(\omega)^H \vec{S}_{m,i+1} \qquad (12)$$

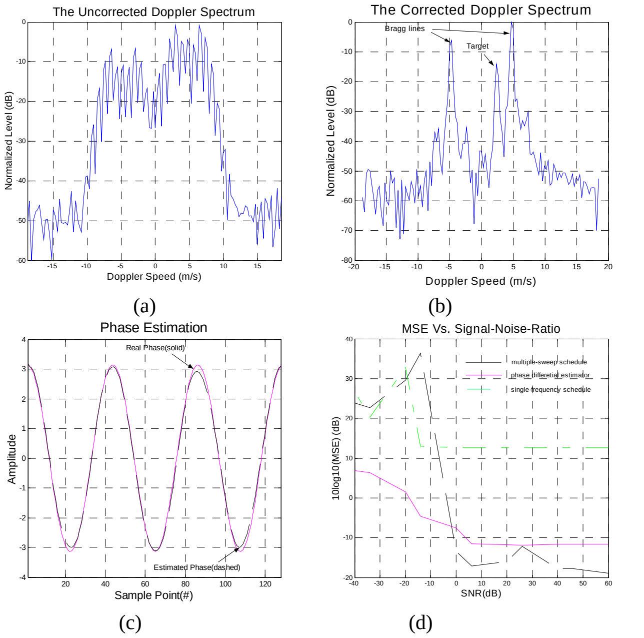
where $\vec{S}_{m,i+1}$ is the signal subspace vector of the (i+1)-th sub-interval, $G_{n,i}$ is the noise subspace vector of the i-th sub-interval, $r$ is the dimension number (i.e. the rank of $R_i$) determined by the previous step and $1 \le i \le Q-1$. When $\mu_i(\omega)$ gets to its minimum, the associated value of $\omega$ is the estimation of the differential radius frequency of the neighboring sub-intervals. And the differential frequency is

$$\Delta f_i = \frac{1}{2\pi} \min_{\omega} \left(\mu_i(\omega)\right) \qquad (13)$$

For better accuracy, the forward and backward sliding window averaging method can be used to process the available data.

When the quasi-instantaneous frequency difference of the phase contamination function $\Delta f_i$ has been estimated, a frequency integration step must be applied to integrate the phase sequence of the correction signal. The estimated phase can be expressed by

$$\hat{\phi}(n) = \exp\left( j \sum_{i=1}^{n} \Delta f_i \right) \qquad (14)$$




(a) (b)

(c) (d)

Fig.2 (a) Doppler spread spectrum contaminated by the sinusoid phase model; (b) Spectrum after the compensation using the conjugate of the estimated contamination function; (c) Original phase of contamination function (solid line) and its estimates (dashed line); (d) Mean square error of different estimation method: the energy-weighting average phase differential estimator (solid line); the eigen-decomposition technique by single sweep frequency scheme (dot-dashed line); the eigen-decomposition technique by multiple-sweep-frequencies scheme (dashed line)

Because of the sub-interval sequence dividing in time domain, an interpolation operation is necessary to recover original sample number. In the experiment, we use the piecewise cubic spline interpolation to reconstruct the phase contamination.

Finally using the conjugate of the estimated correction signal, we can compensate the distortion of the phase path variation due to the ionosphere.

## IV. EXPERIMENT TESTS AND PERFORMANCE ANALYSIS

For experimental purpose, such parameters can be applied as: operating base frequency $f_0 = 10\text{MHz}$, sub-interval number $\text{M} = 8$, sweep frequency difference $df = 60\text{kHz}$, coherent integration time CIT is $51.2\text{s}$, sub-sweep-frequency repetition period (sub-PRI) is $50\text{ms}$, sample point number in a coherent integration time N is 128, sub-intervals number Q is 32, data over-lapped rate is 50%, and sub-sweep-frequency rate $\alpha = 60\text{kHz/s}$.

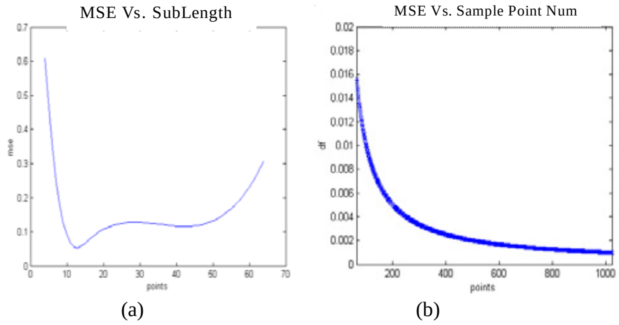
Without loss of the generality, we use a single tone phase signal as the ionospheric phase modulation model. The model is defined as

$$\phi(n) = \beta \sin(2\pi\gamma n), \quad 0 \le \text{n} \le \text{N-1} \qquad (15)$$

Let $\beta = 0.5$ and $\gamma = 3$, and the processing results are shown in Fig.2. Fig.2 (a) illuminates the Doppler spectrum contaminated by the phase model defined in (15). It is clear that due to the Doppler spread of the Bragg lines energy-dominated in spectrum any target can not be resolved from this spectrum. Fig.2 (b) gives the spectrum after correction process with the proposed multiple-sweep-frequencies scheme. Significant spectral sharpening can be observed so that the target echo adjacent to the positive Bragg line is able to be resolved and the original spectrum is restored.

The comparison between the estimated phase and the original phase is shown in Fig.2 (c). The dashed line represents the estimated phase while the solid line represents the original one. We can find that the error between them after the spline interpolation operation, which determines the sharpening degree of spectrum peaks, is acceptable.

Fig.2 (d) shows the estimation performance of different estimation method with the signal-to-noise ratio (SNR). Here SNR is defined as the power ratio between the positive first-order Bragg line and noise, also known as positive-first-order-ocean-clutter-to-noise ratio strictly. The dashed line illuminates the result estimated by the proposed multiple-sweep-frequencies scheme. The dot-dashed line shows the accuracy of the eigen-decomposition technique of single sweep frequency scheme, where the autocorrelation matrix is estimated by the correlation method. The solid line represents the result using the energy-weighting average phase differential estimator given in [6], and also this method applies multiple-sweep-frequencies scheme. It can be concluded that if SNR is large enough, the multiple-sweep-frequencies scheme based on eigen-decomposition technique has better accuracy than other methods. But under a SNR threshold, the performance of the proposed scheme degrades rapidly with the deceasing SNR.




(a) (b)

Fig.3. (a) Mean square error using the proposed scheme with different sub-interval length; (b) Mean square error using the proposed scheme with different sample number.

The estimation accuracy of the phase contamination may depend on some parameters, such as sub-intervals number Q and sample number N. If the coherent integration time is constant, that is the sample point number is not variable in the same sample frequency, the mean square error will arrive the minimum at certain sub-interval length. Fig.3 (a) illuminates the mean square errors with different sub-interval length. From Fig.3 (a), it can be observed that when the length of sub-interval is 16, i.e. the sub-intervals number $\text{Q} = \text{N}/16$ is 64 where the sample length of the time sequence N is 1024 and the data over-lapped rate is 0%, the estimation error reaches its minimum.

If the sampling frequency and the sub-intervals number are constant, a trend that the longer coherent integration time (CIT)

is, the more accurate estimation is can be concluded from Fig.3 (b). However, as the sample point number increases, the computation load also enlarges that may be lower the real-time processing performance of the whole system.

## V. CONCLUSIONS

A set of pulses with different frequencies is applied to construct the 'sweep-frequency' dimension. If the operating frequency of the transmitting pulses changes, the Doppler frequencies of target echo and the Bragg lines should change correspondingly. And the trends of their variations are different, it can be considered that they have different "sweep-frequency angles" in the "sweep-frequency" dimension. Therefore, a full-rank autocorrelation matrix can be achieved. Unlike the energy–weighting phase differential estimator by Parent [6], the proposed method need not apply pre-filtering process before estimation, so the error derived from the process can be avoided.

An operable solution using the segmenting range transform is also addressed to avoid the adjustment of radar system for multiple-sweep-frequencies scheme at the case of estimation of the ionospheric phase path distortion. It is effective under reasonable assumptions and makes the proposed scheme feasible.

It has been shown that if associated parameters are chosen properly, the multiple-sweep-frequencies scheme based on eigen-composition technique can effectively remove the ionospheric phase path contamination. By model experiment verification, achievable estimation accuracy is better than those by other existing methods.

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